International Journal of Theoretical Physics, Vol. 29, No. 1, 1990

Fifth Force as a Manifestation of Torsion

Venzo de Sabbata[1,2] and C. Sivaram[1,3]

Received April 20, 1989

A deviation from the Newtonian inverse square law can arise from torsion coupling.

Recent reanalysis of the Eötvos experiment (Fischbach *et al.*, 1986) has suggested that there might be a deviation from the Newtonian inverse square law of the form

$$V(r) = -G\alpha(m_1 m_2/r)[1+\alpha \exp(-r/\lambda)] \qquad (1)$$

and systematic deviations in the Eötvos experiment suggest that α is negative (i.e., a repulsive component is present) and has a value $\alpha = -7\times 10^{-3}$ with $\lambda \approx 200$ m.

Thus, it has been suggested that there might be a Yukawa-type modification to the inverse square law, at a distance of a few hundred meters. It is interesting that independent geophysical measurements carried out in deep mines over a period of some years also suggest (Stacey *et al.*, 1987; Thieberger, 1987) a possible modification of the Newtonian law to the form given by equation (1), with very similar values for the parameters α and λ.

It is also thought that the additional non-Newtonian component might couple to hypercharge or isospin, i.e., show a composition dependence, and several experiments have been planned to detect this (Stacey *et al.*, 1987; Boynton *et al.*, 1987; Stubbs *et al.*, 1987). Thus, the possibility of a Yukawa-type correction to Newtonian gravity has aroused considerable interest. It must be remarked that there are several theoretical frameworks which can

[1]World Laboratory, Lausanne, Switzerland.
[2]Department of Physics, University, Ferrara, Italy, and Istituto di Fisica Nucleare, Sezione di Bologna, Italy.
[3]Astrophysical Department, Bangalore, India.

0020-7748/90/0100-0001$06.00/00

give rise to and indeed require such modifications. For instance, supergravity theories require, in addition to spin-2 gravitons, adjacent spin partners, i.e., the gravitino and the graviphoton ζ and graviscalar σ. The ζ and σ can receive small masses from radiative corrections, the Higgs mechanism, for instance, giving rise to the vector particle of $m_V \approx g_V\langle\phi_M\rangle$, where $g_V = \chi m_{H}g$ ($\chi = \sqrt{32\pi G}$) and $m_H \sim \langle\phi_H\rangle \sim \langle\phi_M\rangle \sim 1$ TeV, the mass of the Higgs boson, so that $m_V \approx 10^{-8}$ eV, $\lambda = 10$ m. Similarly for σ.

The ζ can give rise to a repulsive force of the type $mm'gg'r^{-1}\exp(-m_\zeta r)$ and σ to an attractive force of $mm'\lambda^2 r^{-1}\exp(-m_\sigma r)$. So one has fifth and sixth forces!

In the $N = 8$ supergravity theory, both ζ and σ are present, while $N = 2$ supergravity has only ζ (no sixth force!). The vector coupling gives a factor depending on the quark composition, i.e., on the neutron–proton ratio of the material.

Again the model of Fujii (1972) requires a dilation and this has a finite range given by the product of the Regge slope and the gravitational constant, i.e., of a few kilometers.

Moreover, gravitational Lagrangians which involve terms quadratic in the curvature, in addition to the usual Hilbert term, which is linear in the curvature, give rise to field equations whose solutions for the field of a point mass involve Yukawa terms (Stelle, 1977) along with the normal $1/r$ potential. That is, for a Lagrangian of the type

$$\int (\alpha_3\chi^{-2}R - \alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu})\, d^4x$$

the corresponding linearized field equations have, for a point mass source, the solution

$$\Phi = -A/r + (B/r)\exp(-\lambda_1/r) - (C/r)\exp(-\lambda_2/r) \tag{2}$$

where

$$A = \chi^2 M/8\pi\alpha_3, \qquad B = \chi^2 M/6\pi\alpha_3, \qquad C = \chi^2 M/42\pi\alpha_3$$

$$\lambda_1 = \alpha_3^{1/2}(\alpha_2\chi^2)^{-1/2}, \qquad \lambda_2 = \alpha_3^{1/2}[2(3\alpha_1 - \alpha_2)\chi^2]^{-1/2}$$

The necessity for such terms arises from quantum gravitational effects as well as arguments based on scale invariance and the renormalizability of gravitational interactions.

A similar situation holds in the case of theories which involve terms quadratic on torsion and curvature in addition to the usual action of the Einstein–Cartan theory (Hehl *et al.*, 1980). It seems natural to introduce gravitational Lagrangians quadratic in torsion Q and curvature R because such squared terms arise in supergravity models and appear in the zero-slope limit of string theories (Candelas *et al.*, 1985). In such a case the torsion becomes a propagating field (Hehl *et al.*, 1980), and in the weak static limit one finds an effective potential with a linearly rising part, i.e., a "confinement potential" of the type $\Phi \alpha r$. That is, for a Lagrangian like

$$L_G \approx (1/4)(\det e^i{}_k)^{-1}[R^i{}_{jkl}R_i{}^{jkl} + Q^i{}_{jk}Q_i{}^{jk}] \tag{3}$$

(with the potential corresponding to the local affine connection $A_\mu{}^{ij}$ and tetrad $e^i{}_k$), the field equations are

$$R^{ijkl}{}_{;l} - (1/2)Q^k{}_{el}R^{ijel} - (1/2)Q^l{}_{el}R^{ijke} = (I_Q)^{kij} \tag{4}$$

Note the curvature–torsion coupling in the field equation (4).

When linearized, the equations are of the fourth order, i.e., of the form $\nabla^4\Phi = \chi m\delta^3(r)$, with a solution $\Phi = \text{const} \cdot r$. However, the weak-field limit is no longer valid for such large values of r and one must go to the so-called "translational gauge limit" characterized by vanishing curvature and non-vanishing torsion which propagates. On the other hand, field equations following from actions involving linear terms in the curvature and torsion in addition to quadratic terms contain, in the linearized static case, mixed terms of second and fourth order, i.e., $\alpha\nabla^4\Phi + \beta\nabla^2\Phi = \chi m\delta^3(r)$, which have solutions involving a $1/r$ and Yukawa terms.

Now in terms of the tetrad frame $e^a = e_\mu{}^a\, dx^\mu$ and the connection $A^{ab} = A_\mu{}^{ab}\, dx^\mu$, the torsion Q^a and curvature R^{ab} can be defined as

$$\begin{aligned} Q^a &= dV^a + A^a{}_b \wedge e^b \\ R^{ab} &= dA^{ab} + A^a{}_c \wedge A^{ab} \end{aligned} \tag{5}$$

We can take a general action with both linear and quadratic terms of the type with local Lorentz invariance:

$$\begin{aligned} &\int (1/4k)[\alpha_0 R^{ab} \wedge^* e^{ab} + \alpha_1 Q^a \wedge^* Q_a + \alpha_2 Q^a \wedge e^b \wedge^* (Q_b \wedge V_a) \\ &\quad + \alpha_3 Q^a \wedge V_a \wedge^* (Q^b \wedge V_b)] + \int \alpha_4 R^{ab} \wedge^* R_{ab} \end{aligned} \tag{6}$$

α_0, α_1, α_2, α_3, and α_4 are suitable constants.

Field equations are obtained by independent variation of the action with respect to e^a and A^{ab}, and in the weak-field limit, putting as usual

$$e^a{}_\mu = \delta^a{}_\mu + (1/2)h^a{}_\mu; \qquad |h^a{}_\mu| \ll 1, \qquad |V\chi A^{ab}{}_\mu| \ll 1 \tag{7}$$

neglecting terms of order h^2, A^2, etc. The metric tensor is given by

$$g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab} = \eta_{\mu\nu} + h_{(\mu\nu)}; \qquad h_{\mu\nu} = h_\mu{}^a \eta_{ab} \delta_\nu{}^b \tag{8}$$

Following Hehl *et al.* (1980), we calculate the effective potential in the static case (by taking the trace of the field equations from the e variation; see also Kim and Yoom (1987).

For $\alpha_0 = 1$ and other terms zero, equation (6) just becomes the Einstein-Cartan Lagrangian, and we have $\nabla^2\Phi = (1/2)\chi\rho$, the usual Poisson equation; for $\alpha_0 = 0$, we have only quadratic terms, giving rise to a confining potential. In the general case the solution is

$$\Phi = -G\alpha(m/r)[1+\alpha \exp(-r/\lambda)] \tag{9}$$

where

$$\alpha = [(2\alpha_0 - \alpha_1 - 2\alpha_2)/(\alpha_1 + 2\alpha_2)][(\alpha_0 + \alpha_1 - 2\alpha_2)/(-\alpha_0 + \alpha_1 - 2\alpha_2)] \tag{10}$$

with a similar expression for λ.

However, it has been observed (Rauch, 1982*a*,*b*; Rauch *et al.*, 1982) that the gravitational theory with the Lagrangian $\sqrt{-g}(-\lambda R + \gamma R^2)$ is *equivalent* to the Einstein-Cartan theory in the presence of a traceless energy-momentum tensor and an arbitrary spin-angular-momentum tensor. Rauch (1982*a*) showed that fluctuations in the scalar curvature can act as a source of torsion due to the presence of terms of the form ∂R; even in the absence of pin, torsion need not vanish. In this sense, torsion is self-generated by the geometry, i.e., by matter-induced fluctuations in the geometry. This is a distinctive feature of $R + R^2$ theories in general (Rauch, 1981; Rauch, 1982*a*,*b*; Rauch *et al.*, 1982). But we know that a theory with both R and R^2 terms has as solution a Newtonian potential plus a Yukawa term. So such potentials can arise in a pure Einstein-Cartan theory. In both cases (propagating and nonpropagating torsion) we have a solution of the form of equation (9). In fact, it was shown (de Sabbata *et al.*, 1989) that in a gauge theory of the local Lorentz group (giving torsion), the gauge-covariant Dirac equation gets additional terms in the Lagrangian of the form

$$-(1/2)\partial_\mu a_\nu \partial^\mu a^\nu - (3/16)g\bar{\psi}\gamma^\mu\gamma_5\psi a_\mu - (3/16)g^2(a^\mu a_\mu)^2 \tag{11}$$

The first term is the kinetic term for the axial vector field, the third a kind of mass term (the second is an interaction term). So if we consider the first and third terms, this is the equation for a massive vector field. In the nonrelativistic limit, such terms were shown to give rise to a interaction between particles with spin (or isospin) which have a g^2/r behavior. With the third term in (11) being considered a mass term, this would have a $e^{-\lambda/r}/r$ behavior.

We can get some limit on the mass m of the particles (bosons) associated with this field from cosmological considerations. Their density should not dominate cosmological dynamics. The exclusion principle would then limit their background density to (also from phase space considerations, i.e., they cannot be distributed such that the mutual separation is smaller than $\hbar/mc$):

$$\rho_b \approx m^4 c^3/\hbar^3 \tag{12}$$

(ρ_b is the background density). Equating $\rho_b \leq \rho_{\text{crit}} \approx 3H^2/8\pi G$ (where ρ_{crit} is the critical density and H the Hubble constant), this would constrain $m \leq 10^{-9}$ eV, giving a range of $\lambda \approx 10^4$ cm i.e., 10^2 m.

As regards the coupling constant αG, if we associate torsion with this extra component, it appears that one can give some evaluation of α. In fact, the effect of torsion of a spinning body is to give rise to a sort of magnetic type of field of the form (de Sabbata, 1988)

$$\bar{B} = (\alpha G)^{1/2}\bar{\sigma} \tag{13}$$

where σ is the spin density and α is the fine structure constant. So this acts as an additional source of the gravitational field, i.e., as a density $B^2 \sim \alpha G\sigma^2$ with a coupling of $\lambda\alpha G$, where $\lambda = \pm 1$. As vector fields with like charges between particles (or between antiparticles) are repulsive, so for particles with isospin of the same sign the force would be repulsive ($\lambda = -1$), and we can say that between particles of opposite isospin (i.e., particles and antiparticles) it is attractive ($\lambda = +1$). If the torsion is propagating, we would also get a Yukawa type of potential as seen from equation (9). So the final form will be of Yukawa type with a coupling of αG, where in that case α is the fine structure constant $1/137 \approx 7\times 10^{-3}$. So far the value of the coupling constant α of the additional term has no theoretical justification. The basis for this value $\alpha \approx -7\times 10^{-3}$ is just from experiments (i.e., reanalysis of the Eötvos experiment, deep mine measurements, etc.), but we notice the curious coincidence that it may be related to the fine structure constant $\alpha = 1/137 \approx 7\times 10^{-3}$.

Thus, such forces coupling to isospin (Isham *et al.*, 1973) can arise in local gauge theories with torsion. For bodies with polarized or aligned spins, this would give rise to a torsion energy term of the form $\bar{S}\cdot(4\pi G/c^2)\sum_i \bar{\sigma}_i$, where $\sum_i \bar{\sigma}_i = (\hbar/2)\sum \bar{n}_i =$ spin density of aligned spins. A polarized macroscopic body should have such an additional energy term. This can induce a torque and may be detectable in a Peres-type experiment (Peres, 1978).

In summary, it may not be necessary to invoke an additional repulsive force if future experiments confirm the need to modify the Newtonian potential. So it turns out that the consequences of torsion for space-time seem to lead to a Yukawa type of additional contribution to the Newtonian potential and this resembles the potential postulated for the fifth force; in the case of rotating bodies, we have some understanding for the coupling strength α to be approximately -7×10^{-3} (between matter and matter).

REFERENCES

Boynton, P. E., *et al.* (1987). *Physical Review Letters*, **59**, 1385.

Candelas, P., Horowitz, G., Strominger, A., and Witten, E. (1985). *Nuclear Physics B*, **258**, 46.

De Sabbata, V. (1988). In *Gravitational Measurements, Fundamental Metrology and Constants* (V. de Sabbata and V. N. Melnikov, eds.), Kluwer Academic, The Netherlands, 115.

De Sabbata, V., and Sivaram, C. (1989). *Nuovo Cimento*, **101A**, 273.

Fischbach, E., *et al.* (1986). *Physical Review Letters*, **56**, 3.

Fujii, Y. (1972). *Annals of Physics*, **69**, 444.

Hehl, F. W., Nistch, J., and von der Heyde, P. (1980). In *General Relativity and Gravitation*, Vol. I (A. Held, ed.) Plenum Press, New York, 329.

Isham, C., Salam, A., and Strathdee, J. (1973). *Nature Physical Science*, **244**, 82.

Kim, J. K., and Yoom, Y. (1987). *Physics Letters*, **A121**, 213.

Peres, A. (1978). *Physical Review D*, **18**, 2739.

Rauch, R. T., and Nieh, H. T. (1981). *Physical Review D*, **24**, 2029.

Rauch, R. T. (1982*a*). *Physical Review D*, **26**, 931.

Rauch, R. T. (1982*b*). *Physical Review D*, **25**, 577.

Rauch, R. T., Shaw, J. C., and Nieh, H. T. (1982). *General Relativity and Gravitation*, **14**, 331.

Scherk, J., and Schwarz, J. (1979). *Physics Letters*, **82B**, 60.

Stacey, F., *et al.* (1987). *Review of Modern Physics*, **59**, 157.

Stelle, K. S. (1977). *Physical Review D*, **14**, 953.

Stubbs, C. W., *et al.* (1987). *Physical Review Letters*, **58**, 1070.

Thieberger, P. (1987). *Physical Review Letters* **58**, 1066.